Discrete Mathematics: Basic Proof Technique for Computer Science Engineering

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Abstract – In this paper we introduce several basic types of proof with special emphasis on a technique called induction that is invaluable to the study of discrete math for computer science.

Index Terms – Discrete Mathematics, Contra positive, Mathematical Induction.

1. INTRODUCTION

Undergraduate Programs in mathematics usually do not include a course in number theory , but student are introduced to elementary number theory concept in other course ,most often in a discrete mathematics course which is usually includes the concept of mathematical induction. Mathematical Induction is a prominent proof technique in discrete mathematics and number theory , where it is used to prove theorem involving properties of the set of natural number

Fermat, the founder of number theory , used a form of mathematical induction to prove many of his discoveries in this fields(Boyer,1968). Beyond its significance as a proof technique in mathematics. [4]

In this section we consider the following general task Given a premise X , How do we show that a conclusion Y hold.

2. DIRECT PROOF

Start with premises X , and directly deduce Y through a series of logical steps.

Claim 1.1: Let n be an integer. If n is even then n^2 is even.

Proof : If n is even then n=2k. for an integer k

$n^2 = (2k)^2 = 4k^2 = 2(2k^2)$ which is even[1,2]

3. INDIRECT PROOF

A Proof by contra positive –start by assuming that the conclusion Y is false, and deduce that the premise X must also be false

Claim 2.1: Let n be an integer , If n^2 is even then n is even

Proof: suppose that n is not even i.e. n is odd

n=2k+1

$n^2 = (2k+1)^2$=4 $k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$, which is odd[1,3]

4. PROOF OF CONTRADICTION

Assume bat the premises X is true and the conclusion Y is false and reach a logical fallacy.

Claim3.1: $\sqrt{2}$ is irrational number

Proof : Assume for contradiction that $\sqrt{2}$ is rational. Then there exist an integer p and q , with no common divisor such

$$\sqrt{2} = \frac{p}{q}$$

$$\frac{p^2}{q^2} = 2$$

$$p^2 = 2q^2$$

This mean is even and by claim 1.1 P is even

Let us replace p by 2k

$$2q^2 = (2k)^2 = 4k^2$$

$$q^2 = 2k^2$$

This time , we conclude that is even is even and q is even as well. But this leads to contradiction. Hence is irrational number[1,3].

5. PROOF BY CASES AND EXAMPLE

Sometimes the easiest way to prove a theorem is to split it into several cases

Claim 4.1: $(n+1)^2 > 2^n$ for every n satisfying $0 \leq n < 5$

Proof: There are only 6 different value of n [1,3]

n	$(n+1)^2$		2^n
0	1	>	1
1	4	>	2
2	9	>	4
3	16	>	8
4	25	>	16

6. NON CONSTRUCTIVE PROOF

This proof does not explicity construct the example asked the theorem , but proves that such an example exist anyways these type of proof are non constructive.

Theorem: There exist irrational numbers x and y such that x^y is rational.

Proof: we know $\sqrt{2}$ is irrational from 2.1

Let Z=$\left(\sqrt{2}\right)^{\sqrt{2}}$

If Z is rational , then we are done (x=y=$\sqrt{2}$)

If Z is irrational, then take x=z=$\left(\sqrt{2}\right)^{\sqrt{2}}$ and y= $\sqrt{2}$ then

$x^y = \left(\left(\sqrt{2}\right)^{\sqrt{2}}\right)^{\sqrt{2}}$=2 is indeed a rational number[1]

7. INDUCTION METHOD

Now with the most basic form of induction over the natural numbers. Suppose we want to show that a statement is true for all natural numbers

e.g. for all n, 1+2+3------n=n(n+1)/2

The basic idea is to approach the proof in two steps to be

1) First Prove that the statement is true for n=1. This is called base case.
2) Next Prove that whenever the statement is true for case n, then it is also true for case n+1. This is called the induction step.

Claim6.1: For every positive integer's n,

1+2+3--------n=$\frac{n(n+1)}{2}$

Proof: Define out induction hypothesis P(n) true if

$$\sum_{i=1}^{n} i = \frac{1}{2}n(n+1)$$

Base case P(1) is clearly true by in section

Inductive step : Assume P(n) is true , we wish to show that P(n+1) is true

$$\sum_{i=1}^{n+1} i = \sum_{i=1}^{n} i + (n+1)$$

$$\frac{1}{2}n(n+1) + n + 1$$

=

$$\frac{1}{2}(n+1)(n+2)$$

This is exactly P(n+1)

Claim6.2: Define our induction hypothesis P(n) to be true if for every finite set S of cardinality

Proof : Base case P(0) is true since the only finite set of size 0 is the empty set , and the power set of the empty set P(={, has cardinality 1.

Inductive step: Assume P(n) is true we wish to show that P(n+1) is true as well. Consider a finite set S of cardinality n+1.

Pick an element e and consider S'=S-{e}

Through the inductive hypothesis

Now consider P(S) observe that a set in P(S) either contain e or not further more , there is a one correspondence between the sets containing e and the sets not containing e.

We have just partitioned P(S) into two equal cardinality subset one of which is P(S') therefore

$$|P(S)| = 2|P(S)| = 2^{n+1}$$

8. CONCLUSION

This paper thus consisted of a brief overview of technique to solve mathematics eq.[1,4].

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